EVOLUTION OF THE DOUBLE-STAR SYSTEMS

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moving with reference to the small stars near it (see *Astronomiche Nachrichten* No. 3143) make it probable that a very slight displacement has occurred; too slight, however, to say that the observed variations are not due to unavoidable errors of observation.

EVOLUTION OF THE DOUBLE-STAR SYSTEMS.

BY DR. T. J. J. SEE, UNIVERSITY OF CHICAGO.

[Abstract of a paper read before the Chicago Academy of Sciences, February 7, 1893.]

Sound cosmogonic speculation begins with KANT, who was the first of modern philosophers to advance a definite mechanical explanation of the formation of the heavenly bodies,* and particularly of the bodies composing the solar system. The views of KANT do not seem, however, to have received much scientific recognition until after LAPLACE'S independent formulation, in more exact mathematical terms,† of a similar explanation of the origin of the planetary system, based upon remarkable phenomena observed in the motions of the planets and satellites, and known as the nebular hypothesis.

Partly on account of the overwhelming‡ argument of LAPLACE in favor of a *natural* or *mechanical* explanation§ of the origin of the planetary system, and the sound dynamical conception underlying the great geometer's hypothesis, and partly on account of the keen interest and speculation arising out of Sir WILLIAM HERSCHEL'S epoch-making investigations of the nebulæ, the nebular hypothesis was soon accepted by astronomers as an explanation entitled to scientific belief. The classic researches of Sir JOHN HERSCHEL tended still further to establish confidence in LAPLACE'S view of the nebular origin of the heavenly bodies; but when Lord ROSSE'S great reflector showed the discontinuous

* See KANT'S *Allgemeine Naturgeschichte und Theorie des Himmels*, published in 1755; *Sämmtliche Werke*, Vol. I, p. 207.

† See *Système du Monde*, Note VII et dernière, p. 498.

‡ See LAPLACE'S remarks in the introduction to his *Théorie Analytique des Probabilités*, p. 67.

§ NEWTON regarded the planets as having been set in their orbits by the immediate hand of the Deity, and held that the fixed stars had been intentionally placed at such vast distances apart in order that they might not fall upon one another by their mutual gravitation. See his remarks in the *Scholium Generale*, p. 527, of Sir WILLIAM THOMSON'S edition of the *Principia*.

nature of some of the objects then classed as "nebulæ," the question arose, whether, with sufficient power, all "nebulæ" might not be resolved into discrete stars. Fortunately, the invention of the spectroscope about 1860, and Dr. HUGGINS' application of it to the study of the heavenly bodies, at once answered this question in the negative, by showing that many of the nebulæ are masses of glowing gas in the process of condensation; and hence it then became a matter of great scientific interest to investigate the formation of the heavenly bodies.

The principlé of the conservation of energy and the mechanical theory of heat, which HELMHOLTZ was the first to apply to the nebular contraction of the Sun,* and LANE'S researches on condensing gaseous masses,† together with the researches of Sir WILLIAM THOMSON on the Sun's age‡ and heat, have each marked important epochs in the development and confirmation of the nebular hypothesis as now maintained and generally accepted by astronomers. The nebular origin of the heavenly bodies being at present generally conceded, the main question of interest relates to the *process* involved in the development of cosmical *systems.*

The nebular hypothesis of LAPLACE supposes the planets and satellites to be the condensed products of rings successively shed by the contracting nebula which originally contained the matter of the solar system, and this theory of ring-formation has exercised extraordinary influence over the minds of scientific men. Prior to the researches of Prof. G. H. DARWIN on the origin of the lunar-terrestrial system, the theory of ring-formation appears never to have been seriously questioned, at least as respects the planetary evolution. But Prof. DARWIN'S discovery of the exceptional formation of the Moon, and his introduction of the important physical agency of tidal friction (which was entirely overlooked by LAPLACE) necessitated considerable modification of the original nebular hypothesis, and constituted, perhaps, the most important step in scientific cosmogony made during this century. Since tidal friction is a necessary adjunct of gravitation wherever systems of fluid bodies exist in a state of relative

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* See the Popular Lecture delivered on the occasion of the KANT Commemoration at Königsberg, February 7, 1854.

† See *American Journal of Science*, July, 1870.

‡ See *Popular Lectures and Addresses* of Sir WILLIAM THOMSON, Vol. I, p. 349.

motion, we perceive that it is a physical agency as universal as gravitation itself, operating more or less powerfully in all the systems of the universe.

It is but proper to state, however, that Prof. DARWIN'S researches on tidal friction were applied only to the solar system, in which the conditions are highly unfavorable to the theory (except in the case of the Earth and Moon), chiefly on account of the relatively small masses of the attendant bodies. In the stellar systems, where each body is sufficiently large to have a considerable moment of momentum of axial rotation, the secular effects of tidal friction must be of far greater importance, and it will therefore not be surprising if we find that this physical agency has played a more prominent part in the development of such systems than even in the case of the Earth and Moon.

It may be remarked that nearly all the cosmogonic speculations hitherto promulgated have been advanced with especial reference to the solar system. For it appears that no systematic investigation of the origin of double stars was ever attempted prior to my own researches, which were begun in an elementary manner about four years ago.

The first step in the investigation was the collection of a table of the best orbits available, which were found to be highly eccentric in comparison with the orbits of the planets and satellites. It was at once evident that so remarkable and fundamental a difference could not be overlooked in explaining the origin of double stars, and the high eccentricities seemed to point with overwhelming probability to the operation of some powerful physical cause which had not left a corresponding impress upon the orbits of the planetary system. Accordingly, it occurred to me that the cause which had elongated the double star orbits might be the secular gravitational reaction arising from tidal friction in the bodies of the stars — an hypothesis that has been confirmed by subsequent mathematical research, in which methods were followed analogous to those employed by Prof. G. H. DARWIN in his graphical history of the system of the Earth and Moon. I had seen no intimation that tidal friction could increase the eccentricity, but soon proved it for the case in which the tides lag (less than 90°), only to discover afterwards that a similar result had been reached by Prof. DARWIN several years earlier,*

* See article "Tides," *Encyclopedia Britannica*, Vol. XXIII, p. 378; also, Prof. DARWIN'S well-known papers in the *Philosophical Transactions and Proceedings of the Royal Society*, from 1878 to 1882.

though it had not been given any particular prominence, and was apparently but little known; hence the discovery that tidal friction could increase the eccentricity was an independent one, since at that time my knowledge of Prof. DARWIN'S work was based upon a review* which gave no account of the secular changes of the eccentricity arising from tidal friction.

In the present discussion of the working of tidal friction, we shall merely present some of the secular effects in an elementary geometrical manner, and at length give a sketch of the process by which double stars arise from double nebulæ.

Self-luminous bodies, such as the Sun† and double stars, are certainly in a fluid state (the term *fluid* being used in the most general sense), and there is reason to believe that the *viscosity* or "stiffness" of the fluid is usually small. Therefore, the tides raised in such masses by the attraction of foreign bodies will not be confined to the surface (as in case of the fluid oceans surrounding the nearly rigid Earth), but will extend throughout the whole mass; such tides are termed *bodily* tides, and it is with them that we are here concerned. Now, imagine a double-star system whose components we shall call respectively *Helios* and *Sol*,‡ each of which is of the same order of mass and same general physical condition as the Sun. Suppose both stars to be spheroids endowed with rotations which are rapid compared to their period of revolution about one another, in the same direction, and about axes nearly perpendicular to the plane of orbital motion.

Let the system be started with the spheroids at a considerable distance apart, so that the attraction of either upon the other becomes practically the same as if the masses were collected at the centers of gravity, and suppose the orbit given a small eccentricity. Then, since the fluid is more or less viscous, the tides raised in either mass by the attraction of the other will lag, and if the viscosity is small the angle of the lag will be only a few degrees. For simplicity, we shall now treat the spheroid *Sol* as having its mass collected at its center of gravity, and examine the effects on the eccentricity arising from the tidal reaction of *Helios*; but it must be remembered that in general the whole

* Miss CLERKE'S *History of Astronomy during the Nineteenth Century*.

† A part of this discussion is reproduced from *Knowledge*, of May, 1892.

‡ These names are chosen to fix the attention upon a system composed of two Sun-like bodies, such as we find in double-star systems.

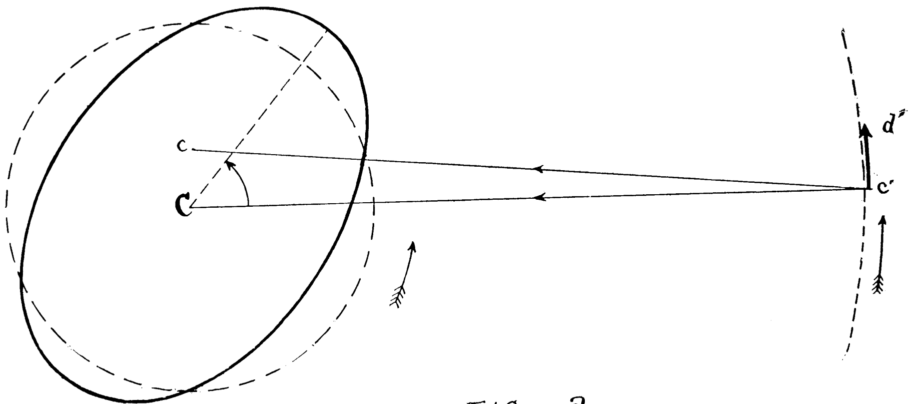

Fig a

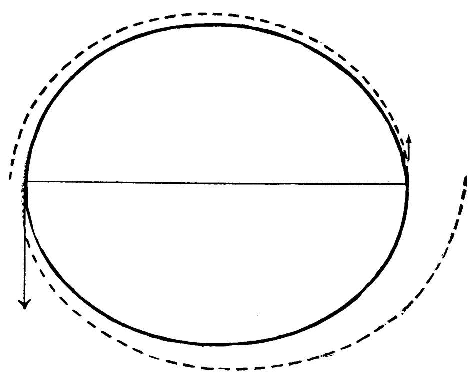

Fig. b.

effect of tidal friction in the system of stars will depend upon the aggregate effect of the double tidal reaction arising from the rotations of both bodies — a complication that renders the rigorous investigation in general very difficult.

With *Sol* thus reduced to a weighted point revolving in the plane of the equator and raising tides in *Helios*, the tidal configuration will be something like that indicated in *Fig. a.*

In the position of the tidal ellipsoid of *Helios* shown in the figure, the whole attraction on *Sol* does not pass through the center of inertia C (about which *Helios* rotates), but through some point c. The reaction of *Sol* is equal and opposite, and hence there arises a couple (with arm cC) acting against the rotation of *Helios*. We may resolve the whole attraction of *Helios* (cc') into two components, one of which ($c'C$) passes through the center of inertia C and produces no effect, as it is counterbalanced by the centrifugal force of the revolving body. The other component ($c'd'$) perpendicular to the radius vector is unbalanced by any opposite force, and hence, acting as an accelerating force, tends to increase the instantaneous linear velocity, whereby there results an increase in *Sol's* mean distance.

As the axial rotation of *Helios* is reduced, *Sol* is wound off on a spiral, whose coils are approximately in the same plane and very close together. To speak mathematically, the *moment* of *momentum* of the whole system is *constant*,* and since the reduction of *Helios'* rotation causes the axial moment of momentum to diminish, it follows that the moment of momentum of orbital motion must augment. In other words, tidal friction transfers moment of momentum of axial rotation to moment of momentum of orbital motion, and hence the mean distance must increase.

With these very brief introductory remarks, let us now examine the changes of the eccentricity of the orbit. In the mathematical works on the tidal theory it is shown that the tide-generating force varies inversely as the cube of the distance of the tide-raising body. The height of the tide varies directly as the tide-generating force. The couple acting against the rotation of *Helios* arises from the excess of the attraction of *Sol* on the nearer tidal protuberance above that on the further. Now, this excess is found to vary inversely as the third power of the distance between the two

* The *energy* of the system, however, is not constant, but continually diminishing, owing to loss of radiant energy.

bodies. But the couple also varies directly as the height of the protuberance (*i. e.*, as the height of the tide), and this height varies inversely as the third power of the distance. Hence, the tidal frictional couple varies as the inverse sixth power of the distance; or it may be described as varying inversely as the square of the tide-generating force, since the tide-generating force varies inversely as the cube of the distance. If we denote the tidal frictional couple by T, the radius vector by ρ, the tangential force by t, the principle of action and reaction gives, for the equilibrium of the forces, $T = t\rho$, or $t = \frac{T}{\rho} = \frac{k}{\rho^7}$, since T varies as $\frac{1}{\rho^6}$. Therefore the tangential disturbing force varies inversely as the seventh power of the distance of the tide-raising body.

When *Sol* is in perihelion the tides are higher (in the inverse ratio of the cube of the distance), and the tangential disturbing force is greater than when *Sol* is in aphelion, in the inverse ratio of the seventh power of the aphelion and perihelion distances. It is well known in the theories of planetary motion that a disturbing acceleration at perihelion causes the revolving body to swing out further than it would otherwise have done, so that when it comes round to aphelion the distance is increased. In like manner, an accelerating force at aphelion increases the perihelion distance, somewhat as we have shown roughly in *Fig. b*.

Now, if we consider the tidal frictional component to act instantaneously and only at the apses of the orbit, the effect would be to increase the perihelion as well as the aphelion distance, but the latter at such an abnormally rapid rate that the orbit becomes more eccentric.*

If the orbit is not very eccentric, similar reasoning to that just employed for the two apses could be applied to other opposite points in the orbit, and the same general result would follow; when, however, the eccentricity is considerable, this method of procedure is not so satisfactory, though while the tides lag, as in *Fig. a*, the eccentricity will continue to increase.

We shall now present the effects of tidal friction as the converse of those arising from a resisting medium, and shall determine the law of the density of the medium required to counteract the effects of tidal friction. Let us consider the case in which the orbit has only a moderate eccentricity (say not surpassing 0.3),

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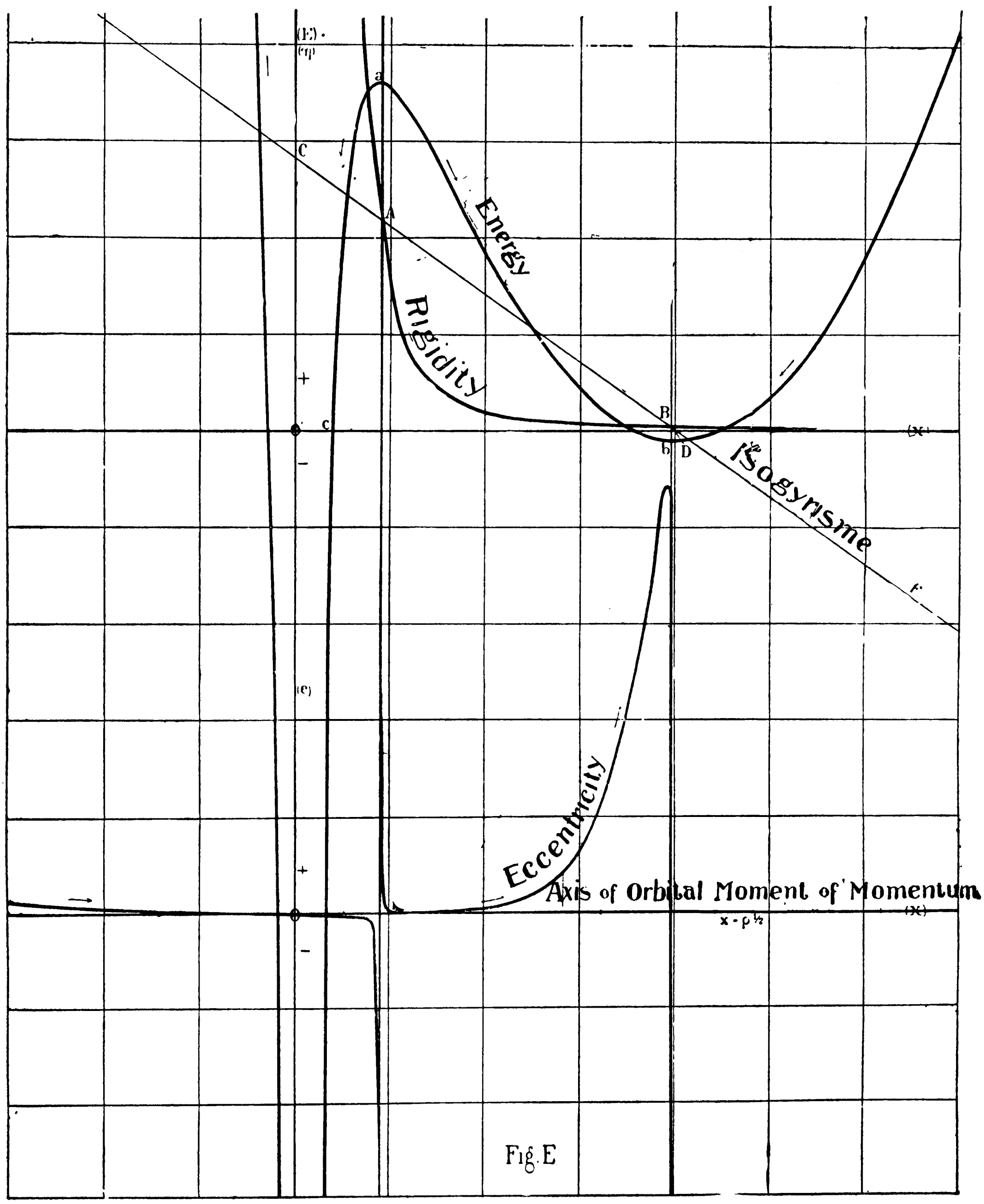
* If the eccentricity is to remain constant, the increase must be in the ratio of $(1-e)$ to $(1+e)$; with tidal friction the ratio is more nearly $(1-e)^7$ to $(1+e)^7$, though not rigorously so except when the eccentricity is very small.



(E)
(η)
a
C
A
Energy
Rigidity
+
c
−
B
b
D
(x)
Isogyrisme
(e)
Eccentricity
Axis of Orbital Moment of Momentum
x - ρ½
(x)

Fig. E

since practically the whole disturbing force due to the tides in *Helios* may then be regarded as acting in the tangent to the orbit. When the tides lag (less than 90°, as in *Fig. a*), the tangential component is directed forward, and hence tends to accelerate the instantaneous linear velocity; the force arising from a a resisting medium is directed continually backward, and hence tends to cause the instantaneous linear velocity to diminish. The two forces are, therefore, oppositely directed, and hence it is evident that if they acted simultaneously the orbit would not undergo the least change either in size or shape, but would be rigorously stable. Now, the resistance encountered at any given point of the orbit depends upon the density of the medium, and is also proportional to the square of the instantaneous linear velocity; but from KEPLER'S law of equal areas in equal times, it follows that the momentary velocity of the revolving body is inversely as the radius vector. The accelerating force due to tidal friction varies inversely as the seventh power of the distance; therefore, in order to counterbalance this by a retarding force due to resistance, we must suppose the density of the medium to vary inversely as the fifth* power of the distance from the center. Such a medium would give a resistance that would just annul the changes arising from tidal friction. Now, LAPLACE has shown † that the action of a resisting medium increasing in density towards the center, according to any law whatever, causes the major axis and the eccentricity of the orbit of a revolving body to diminish. Therefore, tidal friction must cause the major axis and the eccentricity of the orbit to increase.‡

The stellar orbits are on the average about twelve times

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* If σ be the density of the medium, ρ the radius vector, and κ some constant, then the resistance t' varies as $\kappa\sigma v^2$, but v^2 varies as $\frac{1}{\rho^2}$; therefore, t' varies as $\frac{\kappa\sigma}{\rho^2}$. The disturbing force t varies as $\frac{\kappa}{\rho^7}$. But t' must be made equal to t, hence we must suppose σ varies as $\frac{1}{\rho^5}$. Then $t' = t = \frac{\kappa}{\rho^7}$.

† *Mécanique Céleste*, Liv. X, Ch. VII, Sec. 18; or WATSON'S "*Theoretical Astronomy*," p. 552.

‡ We may add that the increase will usually continue until the rotations of both stars are nearly exhausted, after which the eccentricity will be reduced by the libratory motion of the system, and the orbit will at length become circular. The stars, however, would then perhaps be entirely dark, and hence, if in the immensity of space any such dark rigid double-star systems exist, they can not be observed. Other relations of rotation and revolution, and various other viscosities, give rise to various other results; but the conclusion above reached is that of chief interest in connection with the great multitude of double-stars hitherto discovered.

as eccentric as those of the planets and satellites. The mean eccentricity of the 70 orbits now roughly known is 0.45, while the corresponding mean for the orbits of the 8 great planets and their 20 satellites is less than 0.0389. The orbit of γ *Virginis* is known with great precision, and here we have the remarkable eccentricity of 0.9; and the very trustworthy orbit of *Sirius*, recently computed by Dr. AUWERS, has the very considerable eccentricity of 0.63. From a number of other orbits whose eccentricities are very well determined, the fact seems certain that the double-star orbits are generally highly eccentric, though some few appear to be more circular, in accordance with the theory of tidal evolution, under what are perhaps abnormal conditions. Therefore, we have in the general elongation of the double-star orbits a visible trace of the action of secular tidal friction, which has played so important a part in the evolution of the stellar systems, mainly because of the large mass-ratios of the component bodies and their comparative proximity during immense ages; for it must be remembered that double stars, now condensed and widely separated, were millions of years ago much closer together and more expanded in volume, and hence the tidal action was then very much greater than at present. The conclusions here merely stated are confirmed by a rigorous research, which I have recently presented as an "Inaugural Dissertation" to the Faculty of the University of Berlin, and have published under the title "*Die Entwickelung der Doppelstern-Systeme*," to which I must beg to refer all who desire details of the mathematical treatment of the problems arising in double-star evolution.

It now remains to discuss the process by which a nebula under accelerating axial rotation splits up into two comparable masses.

M. POINCARÉ* and Prof. DARWIN† have investigated the equilibrium of rotating masses of fluid with a view of testing LAPLACE'S theory of the formation of the planets and satellites. The researches are widely different in character, but they lead to substantially the same result, namely, that when equilibrium breaks down in a rotating mass the portion detached by increasing angular velocity should bear a far larger ratio to the parent mass than is observed in the planets and satellites of the solar system; and, moreover, that while the separation might ideally

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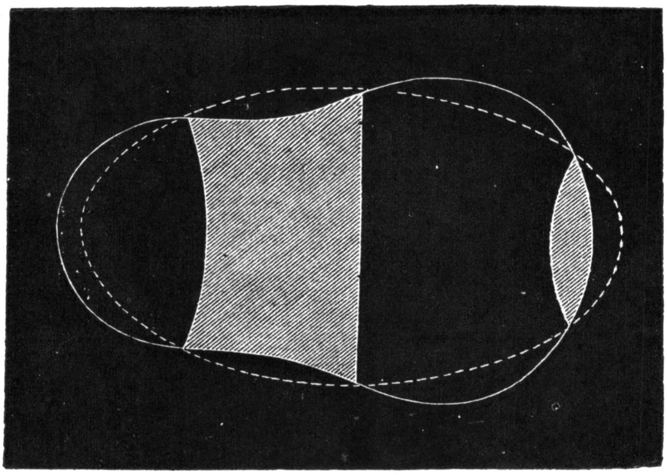
* *Acta Mathematica*, Vol. VII.

† *Phil. Trans.*, 1888.



THE APIOID OF POINCARÉ.

take place in the form of a ring, the general process of division would give rise to masses of a more or less globular form. The Apioid of M. POINCARÉ is given in the accompanying figure, which shows the manner in which the JACOBIAN ellipsoid under increasing axial rotation becomes unstable and finally breaks up into two comparable masses, by a sort of division resembling "fission" among the protozoans. That this process of separation actually occurs in space will be evident on comparing the Apioid with Sir JOHN HERSCHEL'S drawings of double nebulæ, which are here reproduced. It seems legitimate to conclude that double nebulæ have originated from single (perhaps irregular) masses by the process of "fission" arising from increasing rotation, and that in the course of millions of years they will develop into double-stars.

Double nebulæ have been greatly neglected since the time of Sir JOHN HERSCHEL, but it is to be hoped that astronomers will again give adequate attention to these remarkable objects, which should be at once systematically studied and photographed. If accurate drawings or photographs of these objects were now made, it is not to be doubted that important changes could be observed fifty years hence.

Should the theory of double-star evolution here briefly and imperfectly sketched prove to be substantially true, I think it will be conceded that it throws considerable new light upon the problem of the formation of the heavenly bodies. For hitherto nearly all investigators have proceeded in their researches from the point of view of the solar system, notwithstanding the fact that our system is very remarkable, and indeed different from any other hitherto discovered:

(1). *The revolving bodies are very small relative to the central bodies* (except the Moon, whose mass amounts to $\frac{1}{80}$ of the Earth's mass).

(2). *The orbits are nearly circular* (we neglect asteroids and comets).

The double-star systems are remarkable for:

(1). *The large mass-ratios of component bodies.*

(2). *The high eccentricities of the orbits.*

It seems hardly credible, and yet it is a fact, that the Sun has 750 times the mass of all the attendant bodies combined; hence we see that practically all the mass of the solar nebula has gone into the Sun. In double-star systems, the masses, if not equal, are at least comparable. In other words, *the mass-distribution in the solar system is essentially single, whereas in the double-star systems it is essentially double.*

Therefore it is not wonderful that tidal friction has played so prominent a part in the double-star systems, and has been so unimportant in the solar system, where the masses of the revolving bodies are so small as to render their moments of momentum of axial rotation inefficient in changing the size and shape of the orbits. Considering the exceptional character of our system, are we not therefore justified in affirming that the general law of cosmical development can only be deduced from the study of other systems in space, and especially of double stars and double nebulæ, which seem to typify the normal form of celestial evolution? If so, the importance of studying double stars and double nebulæ will be the more easily perceived, as will also the interest attaching to multiple stars and clusters, which deserve the most careful study and the most systematic investigation. For if all the clusters now visible in the heavens were carefully studied and measured by means of photography, it is not to be doubted that in half a century some progress could be made towards explaining the formation of these wonderful bodies, concerning which we are at present profoundly ignorant. If adequate attention is given to other systems in space, we may be sure not only that true cosmogony will be greatly advanced, but that we shall also gain additional light respecting the formation of our own extraordinary system, whose development seems to have been somewhat anomalous. But even in the case of the solar system it is questionable whether the theory of ring-formation is applicable, except in the case of *Saturn's* rings and the asteroids, which appear to have been exceptional formations. The LAPLACEAN theory of ring-formation, although mathematically sound in principle, fails utterly when applied to the actual systems of the universe at large, as we infer from the well-known rarity of ring nebulæ and the great abundance of double nebulæ and double stars. It is to be remarked, however, that it was not known in the time of LAPLACE that a rotating mass of fluid could assume any other than symmetrical figures of equilibrium (including, of course, the *annular form*); but from the researches of POINCARÉ and DARWIN we infer that unsymmetrical figures, such as we observe in double nebulæ, are not only ideally possible, but are in general actually realized in nature. Therefore, since the planets also could have separated in the form of globular masses, there is no longer any logical reason for holding the theory of ring formation, except in the case of *Saturn's* rings and the asteroids, which appear to have been exceptional.

There are other nebulæ worthy of study, particularly the spiral nebulæ, but since their true figures remain uncertain, they have not been considered in this discussion. If adequate attention is given to double, multiple and spiral nebulæ, future research will throw light upon problems which now remain obscure, and in the course of time we shall perhaps be able to reach a definite conclusion respecting the formation not only of our own system, but of systems generally. And when sufficient data have been collected to throw light upon the results of theory, cosmogony ought to rise from the plane of mere speculation to the rank of a real science. If we shall at present succeed in discovering the law of double-star formation, no inconsiderable advance will have been made in the right direction.

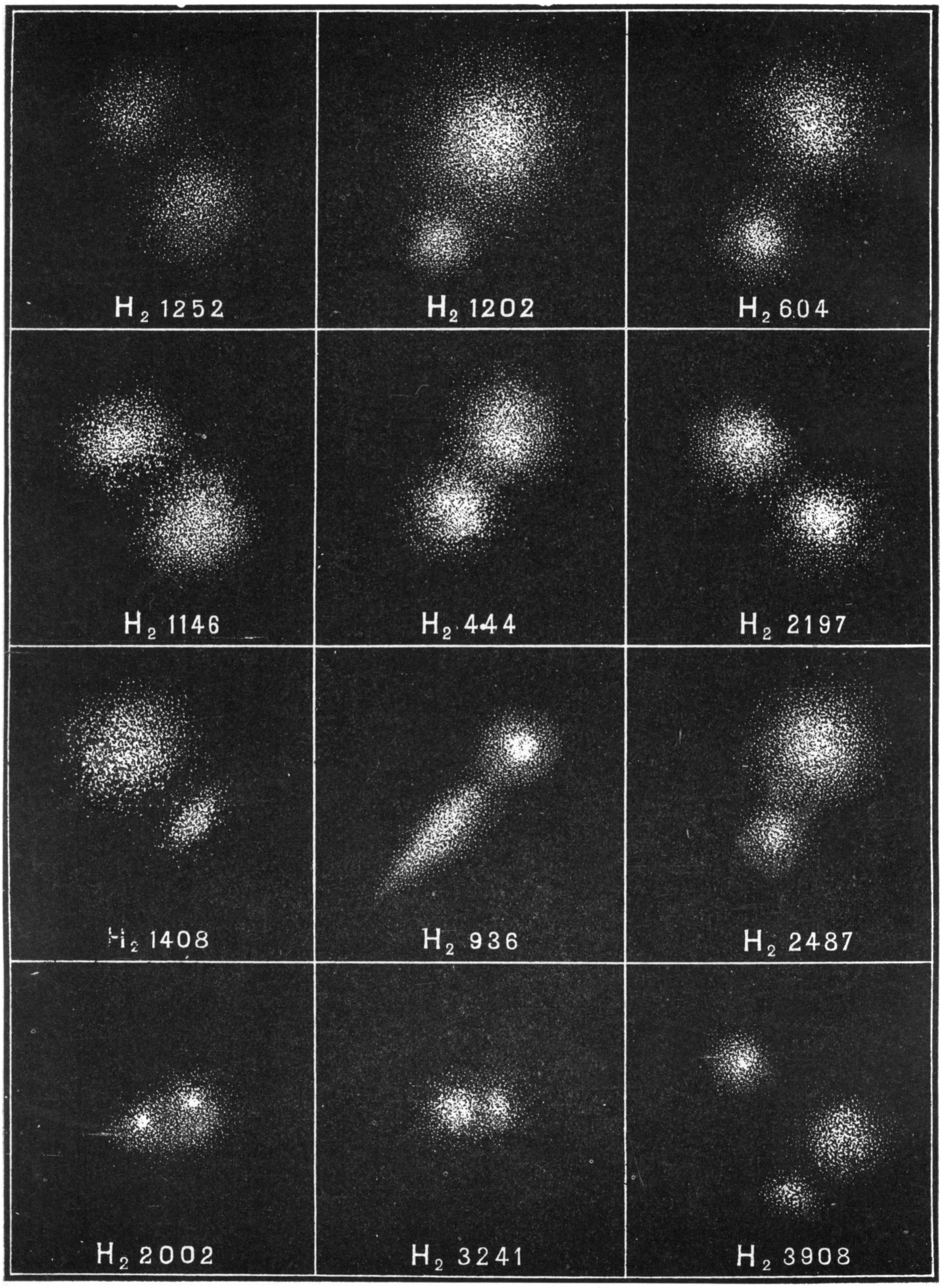
February 21, 1893.

TWO NEW PLANISPHERES.

By W. J. Hussey.

Recently two new planispheres have been issued in this country, one by The Register Publishing Company, Ann Arbor, Mich., and the other by Poole Bros., Chicago. The former was arranged by Prof. Harrington, the latter by Mr. Jules A. Colas. The prices are $1.00 and $3.00 respectively.

The first is much the smaller and less pretentious of the two. It consists of the usual parts, a substantial movable disc, having the constellations mapped upon it, mounted on a square block of heavy cardboard, nearly nine inches on a side. The constellations given include those having north polar distances of less than 120°, and the stars, those of the four brightest magnitudes. A few of the most conspicuous nebulæ and clusters are also given. The disc may be readily set to show that part of the celestial sphere above the horizon at any hour of the night on any day of the year. A key to the positions of the planets is given on the back. This is good till 1901.



HERSCHEL'S DRAWINGS OF DOUBLE NEBULÆ.